2016 IEEE 55th Conference on Decision and Control (CDC)
ARIA Resort & Casino
December 12-14, 2016, Las Vegas, USA

Ensuring Communication Connectivity in Multi-agent Systems in the Presence of Uncooperative Clients

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Abstract— **In this work, the problem of maintaining and guaranteeing communication connectivity between a pair of "client" agents via controlling a number of "router" agents is considered. It is assumed that agents satisfy quadrotor dynamics. A set of controllers are proposed and it is shown that these controllers solve the problem exponentially fast under a set of mild assumptions. The simulation results illustrate the effectiveness of the proposed controllers.**

I. INTRODUCTION

Research in networked multi-agent systems has increased substantially recently due to their low cost in performing tasks. These tasks have a wide range of application from surveillance, exploration to mobile sensor networks and transportation systems. In networked multi-agent systems, an important issue is the communication among agents which is often a necessary requirement for successfully completing the objectives of the system [2]. There are already many research papers considering the problem of ensuring connectivity in multi-agent systems, for example, see [4]–[8]. In many of these works, it is common to assume that all agents cooperate to meet the connectivity requirements.

In many practical scenarios such as tracking fast objects and intercepting intruders not all the agents may coordinate to maintain connectivity. In these situations, a group of agents, termed *clients*, must primarily focus on performing their missions, e.g. they should move to an area of interest for surveillance purposes. For the clients to be able to successfully achieve their objectives, they need to be supported by a second group of agents. An important aspect of such support is ensuring the communication connectivity of the clients. The agents that provide the required support are termed *routers*. In this paper, we focus on designing controllers for the routers so that they ensure the communication connectivity of the clients.

There are some existing results in the literature that address this problem. For example, [9] considers application of aerial vehicles to provide and optimize communication for ground vehicles which are clients. But in the control design stage, the ground vehicles are considered to be static. In [10] considers single-integrator agents with freely moving clients in discrete-time. In this paper, we consider the problem of ensuring the connectivity of two freely moving clients where routers and clients are quadrotors. We note that the clients models here are assumed to be quadrotors for the sake of the simplicity of exposition, however, they can be assumed to have any dynamics as long as their position, velocity, and acceleration can be measured (or estimated) by the routers. Furthermore, we analytically establish that the proposed controllers solve the problem exponentially fast under a set of suitable assumptions.

The paper is organized as follows, in Section II, we provide the problem statement. The controllers and the main results are given in Section III. Simulation results are provided in Section IV to verify the applicability of the proposed controllers and validity of the theoretical results. In the end, concluding remarks and future directions are presented.

II. PROBLEM STATEMENT

We adopt the feedback linearized model of [1] throughout this paper. Consider N quadrotors in $\mathbb{R}^3$ where the feedback linearized model of each quadrotor $i \in \mathcal{V} = \{1, \ldots, N\}$ is given by

$$\dot{x}_i = v_i, \dot{v}_i = -g\bar{c}v_i + \frac{1}{m}F_i, \dot{F}_i = -\beta_q F_i + u_i, \quad (1)$$

where $x_i \in \mathbb{R}^3$ denotes the position of quadrotor i, $v_i \in \mathbb{R}^3$ denotes its velocity, g is the gravity constant, $\bar{c}$ is a constant related to drag force during flight, m is the mass of the quadrotor, $F_i \in \mathbb{R}^3$ is a variable related to lift generated by rotors and attitudes of the quadrotor i, β_q is a constant, and u_i is the control input.

We start by introducing a standard assumption on the maximum communication range of the quadrotors.

Assumption 1: Quadrotors i and j can communicate with each other at time t if $\|x_i(t) - x_j(t)\| \leq R$ where R is a prescribed positive real number that represents the maximum communication range between any pair of agents.
This leads to an undirected connectivity graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$ where $(i, j) \in \mathcal{E}$ if $\|x_i(t) - x_j(t)\| \leq R$. Let $\mathcal{N}_i$ denotes the neighbours of i where $\mathcal{N}_i = \{j | (i, j) \in \mathcal{E}\}$. Moreover, it is assumed that $\mathcal{V}$ is partitioned into two subsets $\mathcal{C}$ and $\mathcal{R}$. The agents in $\mathcal{C}$ are termed *clients* and those in $\mathcal{R}$ are called *routers*.

In broad terms the main objective is to ensure that there is always a path between any pair $i, j \in \mathcal{C}$ via controlling the position of the routers. For the rest of the paper we assume that $\mathcal{C} = \{1, N\}$ and $\mathcal{R} = \{2, \ldots, N-1\}$. The system with two clients is illustrated in Fig. 1. We assume that at time $t = 0$, $\mathcal{N}_i$ satisfies the following assumption.

Assumption 2: Let $\mathcal{M}_i = \{i-1, i+1\}$, for $i = 2, \ldots, N$. Then at time $t = 0$, $\mathcal{M}_i \subset \mathcal{N}_i$, for $i = 2, \ldots, N-1$.

*This work was supported under the Australian Research Council under the Discovery Project DP1094326.

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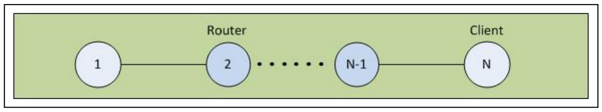

Fig. 1. An example of the multi-agent system considered in this paper.

The objective in this work now is to find u_i for all $i \in \mathcal{R}$ such that the two clients can communicate with each other all the time aided by the routers. In order to achieve the objective, we have the following assumptions on the motions of agents and clients.

Assumption 3: Each agent i measures its own state, i.e. $[x_i^\top, v_i^\top, F_i^\top]^\top$, and receives $[x_j^\top, v_j^\top, F_j^\top]^\top$ from $j \in \mathcal{M}_i$.

Remark 1: Note that if v_j and F_j, $j \in \mathcal{M}_i$, are not directly available then i may estimate them by measuring x_j and receiving u_j instead.

As stated earlier the main problem of interest is to ensure a connected path from client 1 to client N with $N-2$ routers. For this to be feasible we need the following assumption.

Assumption 4: Let $\sup_t \|x_1(t) - x_N(t)\| \leq \delta$, for some bounded real value δ. Then $N - 1 \geq \frac{\delta}{R - \Delta}$, where Δ is a design parameter.

We will provide explicit bounds on Δ later in this paper. The following assumptions provides a limit on the maximum speed of the clients.

Assumption 5: Let $v_{im} = \sup_t \|v_i(t)\|, \quad i \in \mathcal{C}$, then $\max_{i \in \mathcal{C}}(v_{im}) \leq v_{cm}$, where v_{cm} is some bounded positive real value.

In order to guarantee connectivity for all time $t \geq 0$, the initial positions of the agents should satisfy the following assumptions.

Assumption 6: If N is an even number, then it can be written as $N = 2n$ for some integer n. It is assumed that at time $t = 0$ the following statements are true:

$$\|x_n(0) - x_{n+1}(0)\| \leq \frac{1}{3}\|x_{n-1}(0) - x_{n+2}(0)\| \tag{2a}$$

$$\|x_j(0) - x_{2n+1-j}(0)\| \leq \frac{1}{2}(\|x_{j-1}(0) - x_{2n+2-j}(0)\| + \|x_{j+1}(0) - x_{2n-j}(0)\|) \tag{2b}$$

$$\|x_1(0) - x_2(0)\| \leq \|x_2(0) - x_3(0)\| + \frac{1}{\beta}v_{cm} \tag{2c}$$

$$\|x_k(0) - x_{k+1}(0)\| \leq \frac{1}{2}(\|x_{k-1}(0) - x_k(0)\| + \|x_{k+1}(0) - x_{k+2}(0)\|) \tag{2d}$$

$$\|x_{N-1}(0) - x_N(0)\| \leq \|x_{N-2}(0) - x_{N-3}(0)\| + \frac{1}{\beta}v_{cm} \tag{2e}$$

where $k = 2, \ldots, n-1, n+1, \ldots, N-2$, $j = 2, \ldots, n-1$, and β is a positive real number which will be defined later. If N is an odd number, then it can be written as $N = 2n-1$ for some integer n. It is assumed that at time $t = 0$ the following statements are true:

$$\|x_{n-1}(0) - x_{n+1}(0)\| \leq \frac{1}{2}\|x_{n-2}(0) - x_{n+2}(0)\|, \tag{3a}$$

$$\|x_j(0) - x_{2n-j}(0)\| \leq \frac{1}{2}(\|x_{j-1}(0) - x_{2n+1-j}(0)\| + \|x_{j+1}(0) - x_{2n-1-j}(0)\|), \tag{3b}$$

$$\|x_1(0) - x_2(0)\| \leq \|x_2(0) - x_3(0)\| + \frac{1}{\beta}v_{cm}, \tag{3c}$$

$$\|x_k(0) - x_{k+1}(0)\| \leq \frac{1}{2}(\|x_{k-1}(0) - x_k(0)\| + \|x_{k+1}(0) - x_{k+2}(0)\|), \tag{3d}$$

$$\|x_{n-1}(0) - x_n(0)\| \leq \frac{1}{3}\|x_{n-1}(0) - x_{n+1}(0)\| + \frac{1}{3}\|x_{n-2}(0) - x_{n-1}(0)\|, \tag{3e}$$

$$\|x_n(0) - x_{n+1}(0)\| \leq \frac{1}{3}\|x_{n-1}(0) - x_{n+1}(0)\| + \frac{1}{3}\|x_{n+1}(0) - x_{n+2}(0)\| \tag{3f}$$

$$\|x_{N-1}(0) - x_N(0)\| \leq \|x_{N-2}(0) - x_{N-1}(0)\| + \frac{1}{\beta}v_{cm} \tag{3g}$$

where $k = 2, \ldots, n-1, n+1, \ldots, N-1$, $j = 2, \ldots, n-1$, and the same as in the previous case β is a positive real number.

An illustration depicting the distances considered in Assumption 6 can be found in Fig. 2.

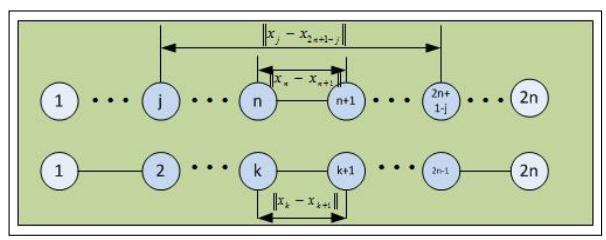

Fig. 2. Initial conditions for x_i when $N = 2n$

Remark 2: Assumption 6 is easily satisfied for small δ as defined in Assumption 4 by making $\|x_i(0) - x_{i+1}(0)\| = \|x_{i+1}(0) - x_{i+2}(0)\|$, $i = 1, \ldots, N-2$.

Then the problem can be formalized as the following.

Problem 1: Consider N agents in set $\mathcal{V}$ where the dynamics of each agent i is governed by (1). Under Assumptions 1–6, find u_i for all $i \in \mathcal{R}$ such that there exists a path between agent 1 and N for all $t \geq 0$.

One way to solve this problem is to guarantee following conditions are satisfied $\forall t \geq 0$.

$$\|x_i(t) - x_{i+1}(t)\| \leq R, \tag{4}$$

where $i = 1, \ldots, N-1$.

Remark 3: The way of solving Problem 1 is equivalent to designing controllers such that the trajectories remain in the non-compact set that characterizes the desired states, i.e. the states that result in the existence of a path connecting 1 and N, if Assumptions 1–6 hold.

III. Controller Design and Analysis for Quadrotor Model

In this section, we first give the designed controller and give the results of connectivity in Theorem 1. In order to prove the theorem, we then give two lemmas, one is on the relationship between an auxiliary system and the original system, the other one is about the connectivity results of the auxiliary system. Finally, the proof of Theorem 1 is given based on the two lemmas.

For a multi-agent system with N quadrotors whose dynamics are as in (1), we design $u_r = [u_2^T, \dots, u_{N-1}^T]^T$ as follows,

$$\begin{aligned} u_i =& \beta_q F_i + m[(-\beta_r + g\bar{c} - 2\beta)a_i + \beta(a_{i-1} + a_{i+1}) \\ &- \beta_s(2v_i - v_{i-1} - v_{i+1})] - \beta_F e_F^i, \end{aligned}$$

where $i = 2, \dots, N-1$, $a_i = \dot{v}_i = -g\bar{c}v_i + \frac{1}{m}F_i$, β_q, β_r, β_s, β_F are positive real constants, $\beta = \frac{\beta_s}{\beta_r}$, $e_F = [{e_F^2}^T, \dots, {e_F^{N-1}}^T]^T$, where,

$$\begin{aligned} e_F^i =& F_i - m[-\beta_r v_i - \beta_s(x_i - x_{i-1}) - \beta_s(x_i - x_{i+1}) \\ &+ g\bar{c}v_i - 2\beta(v_i - \frac{v_{i-1} + v_{i+1}}{2})], \end{aligned}$$

where $i = 2, \dots, N-1$.

By using these control inputs, we have following results regarding the positions of quadrotors.

Theorem 1: Suppose Assumptions 1 – 6 hold. Using controllers u_r in (5), the solution $x_i(t)$ of (1) satisfy the conditions in (4), $\forall t \geq 0$ where $\Delta \geq \frac{2\beta}{N} v_{cm} + 2K_0$ for some positve K_0.

The proof of this theorem is obtianed via applying two lemmas that will be introduced next. The first lemma involves establishing that the solution x_i, $i \in \mathcal{R}$, of multiple quadrotor system (1) for an appropriately chosen set of control laws will converge to the solution of Problem 1 for an auxiliary system comprised of N single-integrators. Then, in the second lemma the connectivity of the clients in the system of single-integrators is demonstrated. The proof of Theorem 1 is then achieved via invoking these two lemmas.

An auxiliary system with N agents with single-integrator dynamics is introduced where

$$\dot{\bar{x}}_i = \bar{v}_i, \tag{5}$$

where $i = 1, \dots, N$, $\bar{x}_i$ is the position of agent i, and $\bar{v}_i$ is its control input. Similar to the assumptions on (1) we assume the following for (5).

Assumption 7: The control inputs $\bar{v}_i$ and states $\bar{x}_i$ in system (5) satisfy Assumptions 1–6 where the agents dynamics are governed by (5) instead of (1).

For this system, we design $\bar{v}_i$, $i \in \mathcal{R}$ as,

$$\bar{v}_i = -\beta(\bar{x}_i - \bar{x}_{i-1}) - \beta(\bar{x}_i - \bar{x}_{i+1}), \tag{6}$$

where $i = 2, \dots, N-1$, β is the same as in (5).

Additionally, it is assumed that the following holds.

Assumption 8: For all $t \geq 0$, $\bar{x}_i(t) = x_i(t)$, $\bar{v}_i(t) = v_i(t)$, $i \in \mathcal{C}$.

Remark 4: This assumption ensures that the clients in the auxiliary system of single-integrators have the same position and velocity of their counterparts in the original system.

We denote

$$e_x = [{e_x^2}^T, \dots, {e_x^{N-1}}^T]^T, \tag{7}$$

where,

$$e_x^i = x_i - \bar{x}_i, \tag{8}$$

where $i = 2, \dots, N-1$. Now we have the following lemma,

Lemma 1: Consider systems in (1) and (5), suppose Assumptions 1 – 8 hold. Using controllers in (5) and (6), the $e_x(t)$ in (7) will converge to 0. Additionally, there exists some $K_0 > 0$, $\gamma > 0$, such that $e_x(t)$ satisfies $\|e_x(t)\| \leq K_0 e^{-\gamma t}, \forall t \geq 0$.

Proof: See the Appendix. ■

In the following Lemma 2, a result of connectivity from the controller (6) designed for system (5) will be stated.

Lemma 2: Suppose Assumption 7, 8 hold for system (5), using controller in (6), the solutions of states $\bar{x}_i(t)$ will always satisfy the following conditions $\forall t \geq 0$.

$$\|\bar{x}_i(t) - \bar{x}_{i+1}(t)\| \leq R - \Delta_1, \tag{9}$$

where $\Delta_1 = \Delta - \frac{2\beta}{N}\|v_{cm}\|$, $i = 1, \dots, N-1$[1].

Proof: See the Appendix. ■

After having the results in the two lemmas, we give the proof of Theorem 1 as follows.

Proof: We already proved in Lemma 2 that the controller designed for single-integrator system (5) will guarantee states $\bar{x}_i(t)$ satisfy the requirements in (9). From Lemma 1, the solution $x_i(t)$, $i \in \mathcal{R}$ of (1) with controller (5) will exponentially converge to $\bar{x}_i$, $i \in \mathcal{R}$. Here we will link these two lemmas to prove the results in this theorem.

From (8), we have, $x_i(t) = \bar{x}_i(t) + e_x^i(t)$, where $i = 2, \dots, N-1$. Based on these equations and Assumption 8, we have,

$$\begin{aligned} \|x_1 - x_2\| =& \|\bar{x}_1 - \bar{x}_2 - e_x^2\| \\ \leq& \|\bar{x}_1 - \bar{x}_2\| + \|e_x^2\|, \end{aligned} \tag{10a}$$

$$\begin{aligned} \|x_i - x_{i+1}\| =& \|\bar{x}_i + e_x^i - \bar{x}_{i+1} - e_x^{i+1}\| \\ \leq& \|\bar{x}_i - \bar{x}_{i+1}\| + \|e_x^i\| + \|e_x^{i+1}\|, \end{aligned} \tag{10b}$$

$$\begin{aligned} \|x_{N-1} - x_N\| =& \|\bar{x}_{N-1} + e_x^{N-1} - \bar{x}_N\| \\ \leq& \|\bar{x}_N - \bar{x}_1\| + \|e_x^{N-1}\|, \end{aligned} \tag{10c}$$

where $i = 2, \dots, N-1$.

From Lemmas 1, for $i = 2, \dots, N-1$, we have,

$$\|e_x^i\| \leq \|e_x\| \leq K_0 e^{-\lambda t}. \tag{11}$$

Based on Lemma 2 and (10),(11), for $i = 2, \dots, N-1$,we have,

$$\|x_i - x_{i+1}\| \leq R - \Delta_1 + 2K_0 e^{-\lambda t}, \tag{12}$$

[1]Note that $\Delta_1 > 0$ because from the statement of Theorem 1 it is known that $\Delta > \frac{2\beta}{N}\|v_{cm}\|$.

By choosing

$$\Delta \geq \frac{2\beta}{N} v_{cm} + 2K_0 e^{-\lambda t}, \tag{13}$$

based on (12), x_i satisfy the conditions in (4). ■

IV. Simulation Results

In the simulation, we choose parameters from [3] in order to have a result close to the practical quadrotor system. These parameters are $g = 9.8, c = 0.01, m = 1.04, \beta_q = 0.1$. For the parameters of controller designed, we set $\beta = 2.5, \beta_r = 5, \beta_F = 5$. R is set as $R = 20$ which satisfies Assumption 1. In the simulation, Assumption 3 holds for we will use these information in the simulation.

The clients are governed by the following inputs:

$$u_i = \alpha \begin{bmatrix} x_i \\ v_i \\ F_i \end{bmatrix} + K(T \begin{bmatrix} x_i \\ v_i \\ F_i \end{bmatrix} - \begin{bmatrix} x_d^i \\ v_d^i \\ a_d^i \end{bmatrix}) + J_d^i,$$

where $i \in \mathcal{C}$, $\alpha = [0_{3\times3}, 0.0098I_3, 0.198I_3]$, $K = [-30I_3, -31I_3, -10I_3]$,

$$T = \begin{bmatrix} 1.04I_3 & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 1.04I_3 & 0_{3\times3} \\ 0_{3\times3} & -0.1019I_3 & I_3 \end{bmatrix},$$

$$x_d^1 = \begin{bmatrix} 10 + 10\sin(\frac{1}{5}t - \frac{\pi}{2}) \\ 10\cos(\frac{1}{5}t - \frac{\pi}{2}) \\ 5 \end{bmatrix}, \quad x_d^N = \begin{bmatrix} 53 + 20\sin(\frac{1}{10}t - \frac{\pi}{2}) \\ 20\cos(\frac{1}{10}t - \frac{\pi}{2}) \\ 5 \end{bmatrix},$$

and v_d^i, a_d^i, J_d^i are the first, second, and third derivatives of x_d^i. Using these u_i, $i \in \mathcal{C}$, and set the initial conditions as $x_1(0) = 0$, $x_N(0) = [33, 0, 0]^T$, $v_i(0) = 0, F_i(0) = 0$, $i \in \mathcal{C}$. We obtain $\sup_t \|x_1(t) - x_N(t)\| = 70$, $\|v_i(t)\| \leq 2.5$, $\forall t \geq 0, \forall i \in \mathcal{C}$. Furthermore, set $\delta = 70$, $v_{cm} = 2.5$ with which Assumption 5 is satisfied, $N = 5$. We set the initial values of clients as $x_4(0) = [\frac{99}{4}\ 0\ 5]^T, x_3(0) = [\frac{33}{2}\ 0\ 5]^T, x_2(0) = [\frac{33}{4}\ 0\ 5]^T$ and $v_i(0) = 0, F_i(0) = 0, i \in \mathcal{R}$, to satisfy the conditions in Assumption 6. From the initial conditions, we have $K_0 = 0$ in (17). And we set $\Delta = 2.5$ as in (13). We can see that these choices satisfy Assumption 4. And in the simulation we label the quadrotors with the number that satisfy the relationships in Assumption 2. With these settings, all the Assumptions 1–6 of this multiple quadrotor system are all satisfied.

The simulation results of distances between neighboring two quadrotors and the norms of v_i are in Fig. 3 and Fig. 4.

From the results, we can see that the distance between any two neighboring quadrotors is less than $R = 20, \forall t \geq 0$. This simulation verify that the controller we designed maintain the connectivity of the multiple quadrotor system, that is satisfy the conditions in (4). Additionally in Fig. 4, we can see the magnitude of velocities of clients can compare to

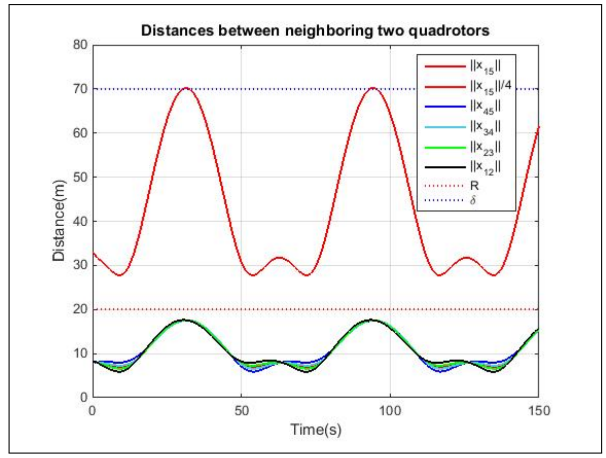

Fig. 3. Distances between two neighboring quadrotors

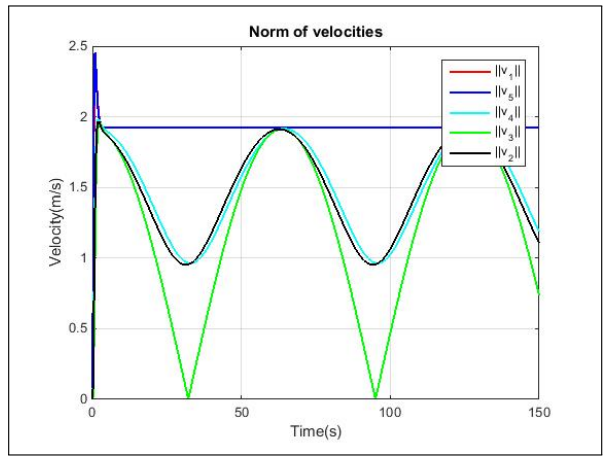

Fig. 4. Norms of velocities

the magnitude of routers' velocities, which implies that our control law can be applied for maintaining connectivity for comparable fast clients.

V. Conclusion and Future Directions

In this work, we consider a practical multi-agent system and design controllers for routers to maintain the connectivity of the system. The main concern of this connectivity design is to guarantee that the clients can move freely to perform their own tasks, which is always the situation in practical environments. The controllers designed for routers are proved to maintain communication connectivity assisted by an auxiliary single-integrator system and the simulation results verified the efficiency of the designed controllers. However in some cases, not all the routers are needed to provide connectivity if a subset of them is sufficiently to maintain connectivity, this is also an interesting aspect to consider.

In this work, we only considered a simple situation with two clients. However, in many situations in practical scenarios, the number of clients are more than two. As a possible future direction, we will extend the results of this paper to design similar controllers for the scenarios where the connectivity of more clients needs to be guaranteed.

Moreover, the disk model of communication with radius R is just an approximation of communication, more realistic communication models need to be considered for connectivity purpose.

APPENDIX

A. Proof of Lemma 1

Proof: By using the control input u_r as in (5), we can calculate the derivatives of e_F in (5) as,

$$\dot{e}_F = -\beta_F e_F. \tag{14}$$

Now the dynamics of the system (1) becomes,

$$\begin{aligned}\dot{x}_i =& v_i,\\ \dot{v}_i =& -\beta_r v_i - \beta_s(x_i - x_{i-1}) - \beta_s(x_i - x_{i+1})\\ &- 2\beta(v_i - \frac{v_{i-1} + v_{i+1}}{2}) + \frac{1}{m}e_F^i,\\ \dot{e}_F =& -\beta_F e_F,\end{aligned}$$

where $i = 2, \ldots, N-1$.

Furthermore, we define e_v as $e_v = [e_v^{2^T}, \ldots, e_v^{N-1^T}]^T$, where, $e_v^i = v_i + \beta(x_i - x_{i-1}) + \beta(x_i - x_{i+1})$, where $i = 2, \ldots, N-1$. By taking derivate of e_v and with the system (1), we have the dynamics of e_v as, $\dot{e}_v^i = -\beta_r e_v^i + \frac{1}{m}e_F^i$, that is,

$$\dot{e}_v = -\beta_r e_v + \frac{1}{m}e_F \tag{15}$$

Then the dynamics of the system can be rewritten as, $\dot{x}_i = -\beta(x_i - x_{i-1}) - \beta(x_i - x_{i+1}) + e_v^i$, $\dot{v} = -\beta_r e_v + \frac{1}{m}e_F$, $\dot{e}_F = -\beta_F e_F$, where $i = 2, \ldots, N-1$.

According to Assumption 6 and dynamics in the above equations and (5), we can taking derivatives of e_x in (7) and get the dynamics of it as, $\dot{e_x} = Ae_x + e_v$, where $A = \begin{bmatrix} -2\beta & \beta & \ldots & \ldots & 0\\ \beta & -2\beta & \beta & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ 0 & \ldots & \ldots & \beta & -2\beta \end{bmatrix}$.

Then the dynamics of e_x, e_v and e_F can be written as,

$$\begin{bmatrix} \dot{e}_x\\ \dot{e}_v\\ \dot{e}_F \end{bmatrix} = \begin{bmatrix} A & I_{N-2} & 0_{N-2}\\ 0_{N-2} & -\beta_r I_{N-2} & \frac{1}{m}I_{N-2}e_F\\ 0_{N-2} & 0_{N-2} & -\beta_F I_{N-2} \end{bmatrix} \begin{bmatrix} e_x\\ e_v\\ e_F \end{bmatrix}. \tag{16}$$

We can see that A is a tridiagonal matrix and it is also Toeplitz, the eigenvalues of A are

$$\lambda_i = -2\beta + 2\beta cos(\frac{k\pi}{N-1}), for\ k = 1, 2, \ldots, N-2$$

which are all negative. So A is Hurwitz. Furthermore, $-\beta_r I_{N-2}$ and $-\beta_F I_{N-2}$ are all Hurwitz, then the system (16) is exponentially stable at the origin. So $e_x(t)$ will converge to 0.

By denoting $x_e = [e_x^T, e_v^T, e_F^T]^T$, we can also conclude that, there exists a $\lambda > 0$, such that $\|x_e(t)\| \leq \|x_e(0)\|e^{-\lambda t}$, $\forall t \geq 0$. By letting,

$$K_0 = \|x_e(0)\|, \tag{17}$$

we have, $\|e_x(t)\| \leq \|x_e(t)\| \leq K_0 e^{-\lambda t}$. ∎

B. Proof of Lemma 2

Proof: The proof will be slightly different between N is even number and odd number. We start the proof from analyzing the system with N is even number, that is $N = 2n$, where $n = 2, 3, \ldots,$. The proof will be similar for $N = 2n-1$, where $n = 2, 3, \ldots,$, so we will just give the results directly.

In the first step of this proof, we check that the distances $\|\bar{x}_i - \bar{x}_{2n+1-i}\|$, where $i = 2, \ldots, N-1$ are less than or equal to some upper bounds which are functions of $\bar{x}_i, i \in \mathcal{V}$. Based on these inequalities, we can find a constant upper bound on $\|\bar{x}_n - \bar{x}_{n+1}\|$. In the second step, we will check that the distances $\|\bar{x}_i - \bar{x}_{i+1}\|$ where $i = 1, \ldots, N-1$ are less than or equal to some upper bounds which are functions of $\bar{x}_i, i \in \mathcal{V}$. By using these inequalities and the upper bound on $\|\bar{x}_n - \bar{x}_{n+1}\|$ in the first step, we can get constant upper bounds on $\|\bar{x}_i - \bar{x}_{i+1}\|$ where $i = 1, \ldots, N-1$, which will satisfy the requirements in (9).

For each pair of $\bar{x}_i$ and $\bar{x}_{2n+1-i}$, we construct a Lyapunov function $V_i = \frac{1}{2}\|\bar{x}_i - \bar{x}_{2n+1-i}\|^2$, where $i = 3, \ldots, n+1$. By this choice of i, we will start the analysis by considering the left-hand part agents in the system as in Fig. 1.

The derivatives of V_i are as follows,

$$\begin{aligned}\dot{V}_n =& (\bar{x}_{n+1} - \bar{x}_{n+2})^T[-3\beta(\bar{x}_n - \bar{x}_{n+1})\\ &+ \beta(\bar{x}_{n-1} - \bar{x}_{n+2})]\\ \leq& -3\beta\|\bar{x}_n - \bar{x}_{n+1}\|(\|\bar{x}_n - \bar{x}_{n+1}\|\\ &- \frac{1}{3}\|\bar{x}_{n-1} - \bar{x}_{n+2}\|),\\ \dot{V}_i =& (\bar{x}_i - \bar{x}_{2n+1-i})^T[-2\beta(\bar{x}_i - \bar{x}_{2n+1-i})\\ &+ \beta(\bar{x}_{i-1} - \bar{x}_{2n+2-i}) + \beta(\bar{x}_{i+1} - \bar{x}_{2n-i})]\\ \leq& -2\beta\|\bar{x}_i - \bar{x}_{2n+1-i}\|(\|\bar{x}_i - \bar{x}_{2n+1-i}\|\\ &- \frac{1}{2}\|\bar{x}_{i-1} - \bar{x}_{2n+2-i}\| - \frac{1}{2}\|\bar{x}_{i+1} - \bar{x}_{2n-i}\|),\end{aligned}$$

where $i = n-1, \ldots, 2$. A sufficient condition under which the derivatives of V_i are less than 0 are as follows,

$$\begin{aligned}\|\bar{x}_n - \bar{x}_{n+1}\| >& \frac{1}{3}\|\bar{x}_{n-1} - \bar{x}_{n+2}\|,\\ \|\bar{x}_i - \bar{x}_{2n+1-i}\| >& \frac{1}{2}(\|\bar{x}_{i-1} - \bar{x}_{2n+2-i}\|\\ &+ \|\bar{x}_{i+1} - \bar{x}_{2n-i}\|),\end{aligned}$$

where $i = n-1, \ldots, 2$. From this condition, we can see the solution of the single-integrator system will converge to states that satisfy the following conditions,

$$\begin{aligned}\|\bar{x}_n - \bar{x}_{n+1}\| \leq& \frac{1}{3}\|\bar{x}_{n-1} - \bar{x}_{n+2}\|,\\ \|\bar{x}_i - \bar{x}_{2n+1-i}\| \leq& \frac{1}{2}(\|\bar{x}_{i-1} - \bar{x}_{2n+2-i}\|\\ &+ \|\bar{x}_{i+1} - \bar{x}_{2n-i}\|),\end{aligned}$$

Additionally, from the requirements on the initial states in (2) in Assumption 2, we can conclude that $\|\bar{x}_i(t) - \bar{x}_{2n+1-i}(t)\|$ will satisfy the former inequalities $\forall t \geq 0$.

Furthermore, by some calculation, we can rewrite the former inequalities as, $\|\bar{x}_i - \bar{x}_{2n+1-i}\| \leq \frac{2n-2i+1}{2n-2i+3}\|\bar{x}_{i-1} - \bar{x}_{2n+2-i}\|$, According to Assumption 4, we have, $\|\bar{x}_1(t) - \bar{x}_N(t)\| \leq (N-1)(R-\Delta)$, $\forall t \geq 0$ Substitute this into the former inequalities, we have, $\|\bar{x}_i - \bar{x}_{2n+1-i}\| \leq (2n-2i+1)(R-\Delta)$. Especially for $\|\bar{x}_n - \bar{x}_{n+1}\|$, we have,

$$\|\bar{x}_n - \bar{x}_{n+1}\| \leq (R-\Delta). \tag{18}$$

In the second step of the proof, we construct another group of Lyapunov functions for each pair of $\bar{x}_i, \bar{x}_{i+1}$, where $i = 1, \dots, n-1$, as $V_{0i} = \frac{1}{2}\|\bar{x}_i - \bar{x}_{i+1}\|^2$, this is still for the left-hand part of the agents in the system as in Fig. 1. According to Assumptions 6 and 3, the derivatives of V_{0i} are as follows

$$\begin{aligned}
\dot{V}_{01} =& (\bar{x}_1 - \bar{x}_2)^T[-\beta(\bar{x}_2 - \bar{x}_1) + \beta(\bar{x}_2 - \bar{x}_3) + v_1] \\
&\leq -\beta\|\bar{x}_1 - \bar{x}_2\|\left(\|\bar{x}_1 - \bar{x}_2\| - \|\bar{x}_2 - \bar{x}_3\| - v_{1m}\right), \\
\dot{V}_{0i} =& (\bar{x}_i - \bar{x}_{i+1})^T[-2\beta(\bar{x}_i - \bar{x}_{i+1}) + \beta(\bar{x}_{i-1} - \bar{x}_i)] \\
&+ \beta(\bar{x}_{i+1} - \bar{x}_{i+2})] \\
&\leq -2\beta\|\bar{x}_i - \bar{x}_{i+1}\|(\|\bar{x}_i - \bar{x}_{i+1}\| - \|\bar{x}_{i-1} - \bar{x}_i\| \\
&- \|\bar{x}_{i+1} - \bar{x}_{i+2}\|),
\end{aligned}$$

where $i = 2, \dots, n-1$.

A sufficient condition under which these derivatives of V_{0i} are less than 0 is,

$$\begin{aligned}
\|\bar{x}_1 - \bar{x}_2\| >& \|\bar{x}_2 - \bar{x}_3\| + \frac{1}{\beta}v_{1m}, \\
\|\bar{x}_i - \bar{x}_{i+1}\| >& \frac{1}{2}(\|\bar{x}_{i-1} - \bar{x}_i\| + \|\bar{x}_{i+1} - \bar{x}_{i+2}\|),
\end{aligned}$$

where v_{1m} is the maximal value of $\|v_1(t)\|$ as in Assumption 3.

Similar as above analysis for V_i, the states $\bar{x}_i$, $i \in \mathcal{V}$ will converge to states such that the following inequalities will always be satisfied.

$$\begin{aligned}
\|\bar{x}_1 - \bar{x}_2\| \leq& \|\bar{x}_2 - \bar{x}_3\| + \frac{1}{\beta}v_{1m}, \\
\|\bar{x}_i - \bar{x}_{i+1}\| \leq& \frac{1}{2}(\|\bar{x}_{i-1} - \bar{x}_i\| + \|\bar{x}_{i+1} - \bar{x}_{i+2}\|),
\end{aligned}$$

where $i = 2, \dots, n-1$.

Additionally, according to (2) in Assumption 5, we conclude that these inequalities are always satisfied $\forall t \geq 0$.

Furthermore, by some calculation, we can rewrite the above inequalities as,

$$\begin{aligned}
\|\bar{x}_1 - \bar{x}_2\| \leq& \|\bar{x}_2 - \bar{x}_3\| + \frac{1}{\beta}v_{1m}, \\
\|\bar{x}_i - \bar{x}_{i+1}\| \leq& \|\bar{x}_{i+1} - \bar{x}_{i+2}\| + \frac{1}{\beta}v_{1m}.
\end{aligned}$$

From former analysis, we have $\|\bar{x}_n - \bar{x}_{n+1}\| \leq (R-\Delta)$ as in (18). By substituting this inequality in the above inequalities, we finally have, $\|\bar{x}_i - \bar{x}_{i+1}\| \leq (R-\Delta) + \frac{n-i}{\beta}v_{1m}$, where $1 \leq i \leq n-1$.

Similar analysis can be applied for the other half side containing agent 1 as in Fig. 1, so we also have, $\|\bar{x}_i - \bar{x}_{i+1}\| \leq (R-\Delta) + \frac{i-n}{\beta}v_{2m}$, where $n+1 \leq i \leq 2n-1$, v_{1m} is the maximal value of $\|v_1(t)\|$ as in Assumption 3.

Analysis on system with $2n$ agents can also be applied similarly on the system consisting of $2n-1$ agents, where $n = 2, 3, \dots$. By using the similar analyzing method and under the conditions of initial states in (3) in Assumption 5, we finally have the following results will always hold $\forall t \geq 0$. $\|\bar{x}_i - \bar{x}_{i+1}\| \leq (R-\Delta) + \frac{2n-2i-1}{2\beta}v_{1m}$, where $1 \leq i \leq n-1$, $\|\bar{x}_i - \bar{x}_{i+1}\| \leq (R-\Delta) + \frac{2i-2n+1}{2\beta}v_{1m}$, where $n \leq i \leq 2n-1$.

We denote d_{imax}, d_{Nmax} as,

$$d_{imax} = \max_i\{\sup_t\|\bar{x}_i(t) - \bar{x}_{i+1}(t)\|\},$$

where $i = 1, \dots, N-1$, and we denote d_{max} as, $d_{max} = \max\{d_{imax}\}$. From Assumption 5, we have, $v_{1m} \leq v_{cm}, v_{2m} \leq v_{cm}$. Then, from the results of upper bound on the distance between neighbouring two agents on systems with odd and even number of agents, we have, $d_{max} \leq (R-\Delta) + \frac{N}{2\beta}v_{cm}$. By denoting, $\Delta_1 = \Delta - \frac{N}{2\beta}v_{cm}$, we have $d_{max} \leq (R-\Delta_1)$. From the definition of d_{max}, we conclude that the states $\bar{x}_i, i \in \mathcal{V}$ satisfy the conditions in (9) $\forall t \geq 0$. ■

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