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Optimal contract design for effort-averse sensors

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ABSTRACT

A central planner wishes to engage a collection of sensors to measure a quantity. Each sensor seeks to trade-off the effort it invests to obtain and report a measurement, against contracted reward. Assuming that measurement quality improves as a sensor increases the effort it invests, the problem of the reward contract design is investigated. To this end, a game is formulated between the central planner and the sensors. Using this game, it is established that the central planner can enhance the quality of the estimate by rewarding each sensor based on the distance between the average of the received measurements and the measurement provided by the sensor. Optimal contracts are designed from the perspective of the budget required to achieve a specified level of error performance.

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1. Introduction

The ubiquity and mobility of networked devices provide unprecedented opportunities for data gathering. The limited authority a central planner may have over the operation of such devices, however, requires careful consideration when the data is used to estimate quantities of interest. In crowd-sensing applications, resource constraints and conflicts of interest arising from privacy concerns, for example, can lead to selfish behaviours in the reporting of data to the central planner. Many studies have speculated on the use of rewards (monetary or otherwise) as a way for the central planner to exert influence over the data gatherers, e.g. Jaimes, Vergara-Laurens, and Raij (2015). If the reward is not tied to the quality of the data provided, effort-averse[1] (or lazy) agents may elect to provide low quality data, instead of expending the effort (e.g. time, energy, loss of opportunity) required to improve the quality.

In general, devising rewards that promote investment of increased effort is complicated by the lack of a priori information about the preferences and sensitivities of individual data gatherers, distribution of the noises introduced by them, and the nature of the variable to be estimated ahead of time. In this paper, to analyse the reaction of the sensors to the rewards, the interaction among the central planner and the sensors is modelled using a game. As a first step towards understanding this problem, it is assumed that the sensitivity of the sensors to payment and the parameters of the sensor's cost functions are common knowledge among all the players, i.e. the central planner and the sensors. If this is not the case, the central planner either needs to enquire about the unknowns or learn them, which opens the door to a higher level strategic behaviour from the sensors. In the case of enquiring about them, mechanism design strategies can be utilised to extract the truthful data (Nisan, 2007). Alternatively, the central planner can employ a policy of the form developed in this paper and gradually change the parameters of the policy to achieve its desired level of performance. However, prior to selecting learning dynamics, the nature of the game equilibria and answers to fundamental questions such as the form of the optimal policy need to be understood. Furthering such understanding is the aim of this technical note.

Consider a group of effort-averse sensors that is employed by a central planner to acquire measurements of a variable. Sensors are rewarded by the central planner to incentivise the acquisition of high quality measurements. The reward contract is fixed before any measurements are taken. The problem investigated in this paper is the design of a reward contract that specifies, ahead of time, how a sensor will be compensated for the measurement it provides. Each sensor responds to the contract by determining the level of effort to invest for acquiring a measurement to report, while striking a balance between the corresponding cost to itself and the expected reward. This interaction between the central planner and the sensors is modelled by a game. When the measurement noise of the sensors and the underlying variable to be estimated are Gaussian, the central planner utilises the optimal estimator (i.e. the least mean square error estimator). However, this estimator is only optimal within the set of linear estimators if the distribution of those variables is not Gaussian (or is unknown with known first and second moments). Furthermore, if the variable to be estimated is deterministic, the previous estimator is not well defined in which case the central planner uses an unweighted averaging estimator. A fundamental property of these estimators is proved in terms of the minimum budget required to achieve a specified level of estimation error. It is shown that a compensation policy based on the second-order empirical statistics of the received measurements can achieve the least budget. The specific form

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of the policy, in addition to the assumption that the measurement noises of the sensors are independent, decouples the best response of each sensor from the actions of the rest. This also ensures that the efforts invested by sensors at the equilibrium of the game are dominant strategies. Hence, it is in the best interest of each effort-averse sensor to expend the equilibrium effort, even when some sensors exhibit non-rational behaviour because of a fault or security breach.

Several studies have proposed bidding mechanisms for recruiting or retaining a useful set of participants while minimising expenditure of the central planner (Duan et al., 2014; Gao, Hou, & Huang, 2015; Hoh et al., 2012; Jaimes, Vergara-Laurens, & Labrador, 2012; Lee & Hoh, 2010; Zhao, Li, & Ma, 2014). In those studies, the sensors are most often assumed to sell homogeneous (interchangeable) sensing data (Jaimes et al., 2012; Lee & Hoh, 2010) and in Duan et al. (2014) the cost of reporting data is considered to be constant. By contrast, the work presented here focuses on influencing the underlying effort invested by the participating sensors in order to improve the quality of estimates determined by averaging. This paper is closer, in essence, to the studies in Yang, Xue, Fang, and Tang (2012), Gao et al. (2015), Zhao et al. (2014), Han, Zhang, Luo, Hu, and Veeravalli (2016), Luo, Tan, and Xia (2014), Koutsopoulos (2013), Chen, Gupta, and Huang (2016), Dobakhshari, Li, and Gupta (2016) and Westenbroek, Dong, Ratliff, and Sastry (2017), where the sensors are assumed to incur a cost that is proportional to the time or the energy dedicated to a sensing task. However, those studies differ in several ways from the framework formulated in this paper. Perhaps most importantly, in the framework developed below the central planner does not measure, inquire, estimate or directly set the effort expended by the sensors in providing measurements. Rather the central planner seeks to influence the effort invested via a pre-determined contract that governs sensor reward. Furthermore, in this paper, more general cost functions are accommodated and optimality with respect to budget constraints is explored more explicitly.

The problem of controlling mobile sensor networks to efficiently allocate their limited resources was also studied in the control community; see, e.g. Zhang (2011),Dames, Schwager, Kumar, and Rus (2012),Pao and Baltz (1999) and Ogren, Fiorelli, and Leonard (2004). Those studies however differ from the presented problem formulation in an important way. The sensors in this paper are not cooperative. That is, they do not follow commands or collaborate with each other to find the best estimation quality. The lack of cooperation also dictates that the sensors (and, if exists, the central planner) cannot measure or estimate the effort or the actions of the other sensors in providing measurements.

The framework of this paper also relates to the problem of designing incentives for agents to truthfully communicate their preferences, most often the parameters of their cost functions, which has a long history in the economics literature on mechanism design; see, e.g. Jackson. In those studies, the central planner wishes to make a social decision after discerning from the agents their fixed and privately held preferences. This differs from the problem considered here in that the central planner does not seek to recover the private information associated with the effort expended by each sensor, which is not fixed and merely serves as an indicator of measurement quality. In fact, the efforts are ultimately functions of the contract set by the central planner to elicit higher quality estimates. More recently, incentivisation or contract design has been also utilised within the engineering community (Bitar, Rajagopal, Khargonekar, Poolla, & Varaiya, 2012; Harirchi, Vincent, & Yang, 2014; Tanaka, Cheng, & Langbort, 2012; Tanaka, Farokhi, & Langbort, 2017). However, these studies focus on enforcing an often unique socially desirable outcome. This paper, on the other hand, is about motivating the sensors to invest the most amount of effort given a certain budget.

In Section 2, the underlying problem is formulated in game-theoretic terms. Contracts are constructed in Section 3. Finally, a numerical example is given in Section 4 and concluding remarks are presented in Section 5.

2. Problem formulation

Consider the case where a central planner wishes to obtain an accurate measurement of a variable $x \in \mathbb{R}$ by obtaining measurements from n sensors .[2] The sensors need to expend effort denoted by $a_i \in \mathbb{R}_{\geq 0}$ to measure the variable x. Here, $\mathbb{R}_{\geq 0}$ denotes the set of non-negative real numbers. The effort is unknown to the central planner. It determines the quality of the corresponding sensor measurement, which is given by

$$y_i = x + w_i,$$

where w_i is a zero-mean random variable with the variance $\mathbb{E}\{w_i^2\} = \eta_i(a_i)$ and $\eta_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is an appropriate mapping that captures the measurement quality return on the effort expended by the sensor. Note that the variable to be estimated and the additive measurement noise are not required to be Gaussian and can be arbitrarily distributed.

Assumption 2.1: $\mathbb{E}\{w_i w_j\} = 0$ *if* $i \neq j$.

Remark 2.1: Note that Assumption 2.1 is satisfied as long as the sensors do not have access to each other's measurements and the estimate constructed by the central planner when reporting their measurements. This is satisfied, in the set-up of this paper, as the sensors are not allowed to renegotiate their contract with the central planner and update their measurements after the estimate constructed by the central planner is revealed.

Assumption 2.2: *The mapping* $\eta_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ *is twice continuously differentiable and strictly decreasing.*

Assumption 2.2 simply states the variance of the noise in the measurement decreases (and thus the quality of the provided measurement increases) with investment of more effort by the sensor. The pay-off to sensor i, for the measurement that it provides to the central planner, is modelled as

$$C_i(a_i, a_{-i}) = \alpha_i p_i - f_i(a_i),$$

where $p_i \in \mathbb{R}_{\geq 0}$ is the unit of compensation offered by the central planner to sensor i, which is stochastic as it depends on all reported measurements, $f_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ determines the cost to sensor i for investing an effort equal to a_i, and $\alpha_i \in \mathbb{R}_{\geq 0}$ is the value-of-compensation[3] from the perspective of sensor i. Note the game-theoretic notation $a_{-i} = (a_j)_{j\neq i}$. It is assumed that the sensors deal with the expected cost, that is, they wish to optimise

$$\begin{aligned}\bar{C}_i(a_i, a_{-i}) &= \mathbb{E}\{C_i(a_i, a_{-i})\}\\ &= \mathbb{E}\{\alpha_i p_i - f_i(a_i)\}\\ &= \alpha_i \mathbb{E}\{p_i\} - f_i(a_i).\end{aligned}$$

Throughout the paper, rewards take the form $p_i = \pi_i(y_1, \dots, y_n)$ for a given contract $\pi : \mathbb{R}^n \to \mathbb{R}^n$. The term contract is used because, while the compensation mapping $\pi : \mathbb{R}^n \to \mathbb{R}^n$ is agreed prior to expending the effort and gathering data, the level of compensation p_i is determined after the sensors report their measurements. That is, when the sensors agree to participate in this process, they are given a contract for their reports in the future. Note that the source of randomness in $C_i(a_i, a_{-i})$ is the measurement noise of the sensors as well as the randomness contained in the variable to be estimated x if it is in fact stochastic (not deterministic). The assumption of dealing with the expectation of the cost is useful/valid if the sensors provide several reports (of the same variable or possibly different variables of interest at different times) under the contract, so that their average returns are well modelled with the expectation of their cost functions.

Assumption 2.3: *The mapping $f_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is twice continuously differentiable and strictly increasing.*

Assumption 2.3 states that the cost of the sensor increases with an increase in the amount of the effort. Therefore, a sensor with limited resources, e.g. time and energy, is reluctant to invest a large effort in gathering a measurement.

Definition 2.1 (Contract Game): A contract game with compensation contract $\pi : \mathbb{R}^n \to \mathbb{R}^n$ is defined as a tuple $(n, (\mathbb{R}_{\geq 0})_{i=1}^n, (\bar{C}_i)_{i=1}^n)$ that encodes n effort-averse sensors each with the action space $\mathbb{R}_{\geq 0}$ and utility $\bar{C}_i(a_i, a_{-i}) = \alpha_i \mathbb{E}\{\pi_i(y_1, \dots, y_n)\} - f_i(a_i)$. The parameters of the game include $(\alpha_i)_{i=1}^n$ as well as the parameters of mappings π, $(f_i)_{i=1}^n$, and $(\eta_i)_{i=1}^n$.

Definition 2.2 (Contract Equilibrium): An action tuple $(a_i^*)_{i=1}^n \in \mathbb{R}_{\geq 0}^n$ constitutes an equilibrium of the contract game if $a_i^* \in \arg\max_{a_i \in \mathbb{R}_{\geq 0}} \bar{C}_i(a_i, a_{-i}^*)$ for all $i \in \{1, \dots, n\}$.

The amount of available information about the problem dictates the estimator structure. If the variable x is a zero-mean Gaussian random variable and the additive measurement noises are Gaussian as well, the central planner can employ the optimal estimator $\hat{x}_{\text{opt}} = \mathbb{E}\{x|y_1, \dots, y_n\}$, which can be rewritten as

$$\hat{x}_{\text{opt}} = \left[1 + \sum_j \frac{V_{xx}}{\eta_j(a_j)}\right]^{-1} \sum_{i=1}^n \frac{V_{xx}}{\eta_i(a_i)} y_i, \tag{1}$$

where $V_{xx} > 0$ is the variance of x. The quality of the estimate, in this case, is given by

$$\mathbb{E}\{(x - \hat{x}_{\text{opt}})^2\} = \left[1 + \sum_j \frac{V_{xx}}{\eta_j(a_j)}\right]^{-1} V_{xx}.$$

If the random variables x and w_i, $i \in \{1, \dots, n\}$, are not Gaussian or have unknown distributions, but the variances are known, then the estimate $\hat{x}_{\text{opt}}$ in (1) is still optimal among the set of linear policies. In the case where x is deterministic, the estimate $\hat{x}_{\text{opt}}$ is meaningless and, thus, the central planner employs a simple unweighted averaging estimator to extract

$$\hat{x}_{\text{ave}} = \frac{1}{n}(y_1 + \cdots + y_n). \tag{2}$$

In the absence of any information and stronger assumptions regarding the nature of the variable-to-be-estimated, a simple estimator, such as the averaging one above, is arguably a sensible option. The quality of the averaging estimate is given by

$$\begin{aligned}\mathbb{E}\{(x - \hat{x}_{\text{ave}})^2\} &= \mathbb{E}\left\{\left(x - \frac{1}{n}(y_1 + \cdots + y_n)\right)^2\right\}\\ &= \mathbb{E}\left\{\left(\frac{1}{n}(w_1 + \cdots + w_n)\right)^2\right\}\\ &= \frac{1}{n^2}\sum_{i=1}^n \eta_i(a_i).\end{aligned}$$

Note that the averaging estimator is in fact unbiased as $\mathbb{E}\{\hat{x}_{\text{ave}}\} = x$ in case of the deterministic variable x. The averaging estimator can also be used in the case where x is stochastic; however, it results in inferior performance and might require a higher budget for achieving the same level of performance.

It is evident that the amount of effort a_i expended by sensor i is, implicitly, a function of the devised compensation contract. Having a fixed compensation policy such that $p_i = c$ for all $a_i \in \mathbb{R}_{\geq 0}$ and all $i \in \{1, \dots, n\}$ is not good. This is because, in that case, $\bar{C}_i(a_i, a_{-i}) = \alpha_i c - f_i(a_i)$ for all $a_i \in \mathbb{R}_{\geq 0}$. Now, by Assumption 2.3, it can be seen that it is in the best interest of the sensor to select $a_i = 0$. This is not the preferable outcome for the central planner in terms of the estimate quality. To fix this issue, it is important that $\mathbb{E}\{p_i\}$ becomes a function of a_i. This is the topic of the paper.

Definition 2.3: A compensation policy is *ex ante* individually rational if $\bar{C}_i(a_i^*, a_{-i}^*) \geq 0$, $1 \leq i \leq n$, for any contract equilibrium $(a_i^*)_{i=1}^n$.

Note that, in a liberal society, individually rational compensation policies must be used as, within such a society, the sensors are free to not participate if there is no hope of receiving compensation in return for efforts. Before stating the problem of interest, it is necessary to define the budget required for implementing a policy $p_i = \pi_i(y_1, \dots, y_n)$, which is given by $B = \sum_{i=1}^n \mathbb{E}\{p_i\}$. Note that B is the expected budget, not to be confused with the actual realisations of budget $\sum_{i=1}^n p_i$.

Problem 2.1: *For given performance level $\epsilon > 0$, find a policy π that minimises the budget B over the set of all ex ante individually rational compensation contracts guaranteeing $\mathbb{E}\{(x - \hat{x})^2\} \leq \epsilon$ where $\hat{x}$ is either $\hat{x}_{\text{ave}}$ or $\hat{x}_{\text{opt}}$.*

Throughout the paper, let the functions $f_i' : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and $f_i'' : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote the first- and the second-order derivatives of $f_i(\cdot)$, respectively. Evidently, these functions exist in light of Assumption 2.3. Similarly, $\eta_i' : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\leq 0}$ and $\eta_i'' : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote the first- and the second-order derivatives of $\eta_i(\cdot)$, respectively, which exists by Assumption 2.2. Again, because of Assumption 2.2, it can be shown that $\underline{\eta}_i \leq \eta_i(a_i) \leq \overline{\eta}_i$ with $\underline{\eta}_i = \lim_{a_i \to \infty} \eta_i(a_i)$ and $\overline{\eta}_i = \lim_{a_i \to 0} \eta_i(a_i)$.

With aforementioned assumptions and definitions in hand, the main results of the paper are established in the next section.

3. Compensation based on empirical statistics

As expected, there is a trade-off between the budget required and the quality of the estimate if *ex ante* individually rational compensation contracts are considered. As a first step, this relationship is explored and a fundamental bound on the required budget for achieving a certain estimation quality is provided. This is first done for the case where x is a random variable and the central planner uses the optimal linear estimator in (1).

Proposition 3.1 (Fundamental Budget Requirement – Optimal Linear Estimator): *For all $\epsilon \geq V_{xx}/[1+\sum_{i=1}^n V_{xx}/\underline{\eta}_i]$, the budget B for implementing any ex ante individually rational compensation contract with estimation quality $\mathbb{E}\{\|x-\hat{x}_{\mathrm{opt}}\|_2^2\} \leq \epsilon$ is lower bounded as*

$$B \geq B_{\mathrm{opt}}^{\min} := \min_{(t_i)_{i=1}^n \in \prod_{i=1}^n [1/\overline{\eta}_i, 1/\underline{\eta}_i]} \sum_{i=1}^n f_i(\eta_i^{-1}(1/t_i))/\alpha_i, \quad \text{(3a)}$$

$$\text{s.t.} \quad \sum_{i=1}^n t_i \geq 1/\epsilon - 1/V_{xx}. \quad \text{(3b)}$$

No finite budget can guarantee estimation quality $\epsilon < V_{xx}/[1+\sum_{i=1}^n V_{xx}/\underline{\eta}_i]$.

Proof: Individual rationality ensures that $\bar{C}_i(a_i^*, a_{-i}^*) \geq 0$, $\forall i$. This implies that $\alpha_i \mathbb{E}\{p_i\} - f_i(a_i^*) \geq 0$ and thus, $B = \sum_{i=1}^n \mathbb{E}\{p_i\} \geq \sum_{i=1}^n f_i(a_i^*)/\alpha_i$. Further, the error at such an equilibrium is given by $\mathbb{E}\{(x-\hat{x})^2\} = [1+\sum_j V_{xx}/\eta_j(a_j^*)]^{-1} V_{xx}$. Let $t_i := 1/\eta_i(a_i^*)$. Evidently, $t_i \in [1/\overline{\eta}_i, 1/\underline{\eta}_i]$ and $\mathbb{E}\{(x-\hat{x})^2\} = [1+\sum_j V_{xx} t_j]^{-1} V_{xx}$. Based on Assumption 2.2, it is known that $\eta_i^{-1}(\cdot)$ exists (albeit only on the domain $[\underline{\eta}_i, \overline{\eta}_i]$). Thus, $a_i^* = \eta_i^{-1}(1/t_i)$ resulting in the inequality $B \geq \sum_{i=1}^n f_i(\eta_i^{-1}(1/t_i))/\alpha_i$. Therefore, the smallest budget is given by $\inf\{\sum_{i=1}^n f_i(\eta_i^{-1}(1/t_i))/\alpha_i \mid \sum_{i=1}^n t_i \geq 1/\epsilon - 1/V_{xx}$ with $t_i \in [1/\overline{\eta}_i, 1/\underline{\eta}_i], \forall i\}$. Note that infimum can be replaced by minimum noting the compactness of the set. ∎

A similar result can be proved for the averaging estimator in (2) for the case where x is deterministic.

Proposition 3.2 (Fundamental Budget Requirement – Averaging Estimator): *For all $\epsilon \geq \sum_{i=1}^n \underline{\eta}_i/n^2$, the budget B for implementing any ex ante individually rational compensation contract with estimation quality $\mathbb{E}\{\|x-\hat{x}_{\mathrm{ave}}\|_2^2\} \leq \epsilon$ is lower bounded as*

$$B \geq B_{\mathrm{ave}}^{\min} := \min_{(t_i)_{i=1}^n \in \prod_{i=1}^n [\underline{\eta}_i, \overline{\eta}_i]} \sum_{i=1}^n f_i(\eta_i^{-1}(t_i))/\alpha_i \quad \text{(4a)}$$

$$\text{s.t.} \quad \sum_{i=1}^n t_i \leq n^2\epsilon. \quad \text{(4b)}$$

No finite budget can guarantee estimation quality $\epsilon < \sum_{i=1}^n \underline{\eta}_i/n^2$.

Proof: The proof follows the same line of reasoning as in Proposition 3.1 with the exception that $t_i = \eta_i(a_i^*)$. ∎

The lower bounds in (3) and (4) require solving potentially non-convex optimisation problems. Under some symmetry conditions, these bounds can be greatly simplified.

Corollary 3.1: *If $\eta_i = \eta$, $f_i = f$, $\alpha_i = \alpha$ for all i, $B_{\mathrm{ave}}^{\min} = nf(\eta^{-1}(n\epsilon))/\alpha$ for all $\epsilon \geq \underline{\eta}/n$ and $B_{\mathrm{opt}}^{\min} = nf(\eta^{-1}(n\epsilon[V_{xx}/(V_{xx}-\epsilon)]))/\alpha$ for all $\epsilon \geq \underline{\eta}/[n+\underline{\eta}/V_{xx}]$, where $\underline{\eta} = \underline{\eta}_i$ for all i.*

Proof: The proof follows from the observation that the solutions of the optimisation problems in (3) and (4) must be symmetric if all their parameters are symmetric. ∎

As expected, for the case where x is stochastic, by using the optimal linear estimator (instead of the averaging one) smaller estimation errors are achievable as the lower bounds for ϵ in Corollary 3.1 demonstrate. Note that

$$\begin{aligned} B_{\mathrm{opt}}^{\min} &= nf(\eta^{-1}(n\epsilon[V_{xx}/(V_{xx}-\epsilon)])) \\ &\leq nf(\eta^{-1}(n\epsilon)) \\ &= B_{\mathrm{ave}}^{\min}, \end{aligned}$$

where the inequality follows since $f(\eta^{-1}(\cdot))$ is a decreasing function and $n(1/\epsilon - 1/V_{xx}) \geq n\epsilon$ (since $V_{xx} > 0$). This clearly makes sense as $\hat{x}_{\mathrm{opt}}$ has the lowest estimation error among the set of linear estimators to which the averaging estimator also belongs. However, in the case where x is deterministic, the averaging estimator is the only option.

The next two theorems provide the solution of Problem 2.1 under some mild conditions for averaging and optimal linear estimators. This is done by showing that a policy that rewards each sensor based on the distance between the average of the received measurements and the measurement provided by the sensor achieves the lower bounds (of the budget) in Propositions 3.1 and 3.2.

Theorem 3.1 (Optimal Contract – Optimal Linear Estimator): *Let the contract game satisfy*

$$\lim_{a_i \to \infty} f_i(a_i) = \infty, \quad \text{(5a)}$$

$$\eta_i''(a_i), f_i''(a_i) \geq 0, \quad \forall a_i \in \mathbb{R}_{\geq 0}. \quad \text{(5b)}$$

Then, the budget-optimal compensation contract, among the set of all ex ante individually rational compensation contracts guaranteeing a performance level $\mathbb{E}\{\|x-\hat{x}_{\mathrm{opt}}\|_2^2\} \leq \epsilon$, for given $\epsilon \geq$

$V_{xx}/[1+\sum_{i=1}^{n} V_{xx}/\underline{\eta}_i]$, *is*

$$\pi(y_1,\ldots,y_n)=\delta_i-\gamma_i\left(-y_i+\frac{1}{n}\sum_{j=1}^{n}y_j\right)^2, \quad (6)$$

where

$$\gamma_i=-\frac{1}{\alpha_i}\left(\frac{n}{n-1}\right)^2\frac{f_i'(\eta_i^{-1}(1/t_i^*))}{\eta_i'(\eta_i^{-1}(1/t_i^*))}, \quad (7a)$$

$$\delta_i=\gamma_i\left(\left(\frac{n-1}{n}\right)^2\frac{1}{t_i^*}+\frac{1}{n^2}\sum_{j\neq i}\frac{1}{t_j^*}\right)+\frac{1}{\alpha_i}f_i(\eta_i^{-1}(1/t_i^*)) \quad (7b)$$

and

$$(t_i^*)_{i=1}^n\in\operatorname*{arg\,min}_{(t_i)_{i=1}^n\in\prod_{i=1}^n[1/\overline{\eta}_i,1/\underline{\eta}_i]}\sum_{i=1}^{n}f_i(\eta_i^{-1}(1/t_i))/\alpha_i \quad (8a)$$

$$\text{s.t.}\quad\sum_{i=1}^{n}t_i\geq 1/\epsilon-1/V_{xx}. \quad (8b)$$

Proof: For a compensation contract of the form (6), by Assumption 2.1, it can be shown that

$$\begin{aligned}\mathbb{E}\{p_i\}&=\delta_i-\gamma_i\mathbb{E}\left\{\left(\frac{1}{n}(y_1+\cdots+y_n)-y_i\right)^2\right\}\\&=\delta_i-\gamma_i\mathbb{E}\left\{\left(-\frac{n-1}{n}w_i+\frac{1}{n}\sum_{j\neq i}w_j\right)^2\right\}\\&=\delta_i-\gamma_i\left(\left(\frac{n-1}{n}\right)^2\eta_i(a_i)+\frac{1}{n^2}\sum_{j\neq i}\eta_j(a_j)\right).\end{aligned}$$

Consequently

$$\begin{aligned}\bar{C}_i(a_i,a_{-i})=\alpha_i\delta_i-\Bigg[\alpha_i\gamma_i\Bigg(\left(\frac{n-1}{n}\right)^2\eta_i(a_i)\\+\frac{1}{n^2}\sum_{j\neq i}\eta_j(a_j)\Bigg)+f_i(a_i)\Bigg].\end{aligned}$$

Therefore, any action tuple $(a_i^*)_{i=1}^n\in\mathbb{R}_{\geq 0}^n$ is a contract equilibrium if and only if a_i^*, $\forall i$, belongs to

$$\mathcal{A}_i=\operatorname*{arg\,min}_{a_i\in\mathbb{R}_{\geq 0}}\left[\alpha_i\gamma_i\left(\frac{n-1}{n}\right)^2\eta_i(a_i)+f_i(a_i)\right]. \quad (9)$$

Now, we show that each $\mathcal{A}_i$ is non-empty and thus there exists an equilibrium. Let $\xi_i:\mathbb{R}_{\geq 0}\to\mathbb{R}$ be such that

$$\xi_i(a_i)=\left[\alpha_i\gamma_i\left(\frac{n-1}{n}\right)^2\eta_i(a_i)+f_i(a_i)\right],\quad\forall a_i\in\mathbb{R}_{\geq 0}. \quad (10)$$

It follows that $\lim_{a_i\to\infty}\xi_i(a_i)\geq\lim_{a_i\to\infty}f_i(a_i)=+\infty$, where the inequality holds, by definition, from the property $\eta_i(a_i)\geq 0$ for all $a_i\in\mathbb{R}_{\geq 0}$. As a result, for all $\Xi\in\mathbb{R}_{\geq 0}$, there exists $A(\Xi)\in\mathbb{R}_{\geq 0}$ such that $\xi_i(a_i)\geq\Xi$ for all $a_i\geq A(\Xi)$. Let $\Xi'\in\mathbb{R}_{\geq 0}$ be an arbitrary real number such that $\Xi'>\xi_i(0)=\alpha_i\gamma_i((n-1)^2/n^2)\eta_i(0)+f_i(0)$. Now, define the set $\Omega:=\{a_i\in\mathbb{R}_{\geq 0}\mid a_i\leq A(\Xi')\}$. Clearly, the minimiser of $\xi_i(\cdot)$ belongs to Ω because $\xi_i(a_i)>\Xi'>\xi_i(0)$ for all $a_i\in\mathbb{R}_{\geq 0}\setminus\Omega$. Hence, $\mathcal{A}_i=\arg\min_{a_i\in\Omega}\xi_i(a_i)$. Noting the continuous function $\xi_i(\cdot)$ attains its minimum over the compact set Ω, it follows that $\mathcal{A}_i$ is non-empty.

For some cases, the construction of $\mathcal{A}_i$ can be simplified. Note that $\mathrm{d}^2\xi_i(a_i)/\mathrm{d}a_i^2\geq 0$ under (5b). Therefore, $\xi_i(\cdot)$ is a convex function. To ensure that the tuple $(a_i^*)_{i=1}^n=(\eta_i^{-1}(1/t_i^*))_{i=1}^n$ is an equilibrium of the game, we select

$$\gamma_i=-\frac{1}{\alpha_i}\left(\frac{n}{n-1}\right)^2\frac{f_i'(\eta_i^{-1}(1/t_i^*))}{\eta_i'(\eta_i^{-1}(1/t_i^*))}.$$

This follows from that

$$\frac{\mathrm{d}}{\mathrm{d}a_i}\left[\alpha_i\gamma_i\left(\frac{n-1}{n}\right)^2\eta_i(a_i)+f_i(a_i)\right]\Bigg|_{a_i=\eta_i^{-1}(t_i^*)}=0.$$

Finally, for the proposed compensation contract in (6) to be *ex ante* individually rational, it must satisfy

$$\delta_i\geq\gamma_i\left(\left(\frac{n-1}{n}\right)^2\eta_i(a_i^*)+\frac{1}{n^2}\sum_{j\neq i}\eta_j(a_j^*)\right)+\frac{1}{\alpha_i}f_i(a_i^*).$$

This is because

$$\begin{aligned}\bar{C}_i(a^*,a_{-i}^*)&=\alpha_i\delta_i-\Bigg[\alpha_i\gamma_i\Bigg(\left(\frac{n-1}{n}\right)^2\eta_i(a_i^*)\\&\quad+\frac{1}{n^2}\sum_{j\neq i}\eta_j(a_j^*)\Bigg)+f_i(a_i^*)\Bigg]\\&=\alpha_i\Bigg[\delta_i-\gamma_i\Bigg(\left(\frac{n-1}{n}\right)^2\eta_i(a_i^*)\\&\quad+\frac{1}{n^2}\sum_{j\neq i}\eta_j(a_j^*)\Bigg)-\frac{1}{\alpha_i}f_i(a_i^*)\Bigg].\end{aligned}$$

Clearly, the smallest possible δ_i must be selected to minimise the budget. In that case, the budget for implementing this policy is

$$\begin{aligned}B&=\sum_{i=1}^{n}\left[\delta_i-\gamma_i\left(\left(\frac{n-1}{n}\right)^2\eta_i(a_i^*)+\frac{1}{n^2}\sum_{j\neq i}\eta_i(a_j^*)\right)\right]\\&=\sum_{i=1}^{n}\frac{1}{\alpha_i}f_i(\eta_i^{-1}(1/t_i^*)),\end{aligned}$$

which is equal to $B_{\text{opt}}^{\min}$ in Proposition 3.1. Therefore, the policy must be optimal. ■

Theorem 3.2 shows that a simple compensation policy based on the second-order empirical statistics is in fact optimal from the perspective of the budget required for achieving a specified level of estimation error.

Remark 3.1 (Tightness of Convexity Conditions): The condition (5b) can be replaced with the tighter condition $f_i''(a_i)\eta_i'(\eta_i^{-1}(t_i^*)) - f_i'(\eta_i^{-1}(t_i^*))\eta_i''(a_i) \leq 0$ for all $a_i \in \mathbb{R}_{\geq 0}$. However, by doing so, it becomes considerably harder to check the validity of the assumption and to interpret it.

Remark 3.2 (Equilibrium versus Dominant Strategy): Note that the best action of sensor i in Theorem 3.2 is independent of the actions of the other sensors. Therefore, $a_i^* = \eta_i^{-1}(1/t_i^*)$ is a dominant strategy for the sensor, which is stronger than an equilibrium (in which the sensors do not deviate given that the others do not deviate as well). This means, even if some sensors are faulty or mistakenly determine their effort, it is in the best interest of each sensor i to expend the effort a_i^*. This behaviour makes the estimator robust to individual corruptions. Note that this observation does not exclude the possibility of the sensors being able to improve their compensation by colluding with each other, which is a topic for future research.

Theorem 3.2 (Optimal Contract – Averaging Estimator): *Let the contract game satisfy the assumptions of Theorem* 3.1. *Then, the budget-optimal compensation contract, among the set of all ex ante individually-rational compensation contracts guaranteeing a performance level* $\mathbb{E}\{\|x - \hat{x}_{\text{ave}}\|_2^2\} \leq \epsilon$, *for given* $\epsilon \geq 1/n^2 \sum_{i=1}^n \underline{\eta}_i$, *is shown in* (6), *where*

$$\gamma_i = -\frac{1}{\alpha_i}\left(\frac{n}{n-1}\right)^2 \frac{f_i'(\eta_i^{-1}(t_i^*))}{\eta_i'(\eta_i^{-1}(t_i^*))}, \tag{11a}$$

$$\delta_i = \gamma_i\left(\left(\frac{n-1}{n}\right)^2 t_i^* + \frac{1}{n^2}\sum_{j\neq i} t_j^*\right) + \frac{1}{\alpha_i} f_i(\eta_i^{-1}(t_i^*)) \tag{11b}$$

and

$$(t_i^*)_{i=1}^n \in \underset{(t_i)_{i=1}^n \in \prod_{i=1}^n [\underline{\eta}_i, \overline{\eta}_i]}{\arg\min} \sum_{i=1}^n f_i(\eta_i^{-1}(t_i))/\alpha_i, \tag{12}$$

$$\text{s.t.} \quad \sum_{i=1}^n t_i \leq n^2\epsilon. \tag{13}$$

Proof: The proof follows from the same reasoning as in Theorem 3.1. ∎

Corollary 3.2: *If* $\eta_i = \eta, f_i = f, \alpha_i = \alpha$ *for all i, under the conditions of Theorem* 3.1, *the budget-optimal compensation contract, among the set of all ex ante individually rational compensation contracts guaranteeing a performance level* $\mathbb{E}\{\|x - \hat{x}_{\text{opt}}\|_2^2\} \leq \epsilon$, *for all* $\epsilon \geq V_{xx}/[1 + \sum_{i=1}^n V_{xx}/\underline{\eta}_i]$, *is*

$$\pi(y_1, \ldots, y_n) = [\gamma(n-1)\epsilon V_{xx}/(V_{xx} - \epsilon) + f(\eta^{-1}(n\epsilon V_{xx}/(V_{xx} - \epsilon)))/\alpha] - \gamma\left(-y_i + \frac{1}{n}\sum_{j=1}^n y_j\right)^2,$$

with $\gamma = -(n/(n-1))^2 f'(\eta^{-1}(n\epsilon V_{xx}/(V_{xx} - \epsilon)))/(\alpha\eta'(\eta^{-1}(n\epsilon V_{xx}/(V_{xx} - \epsilon))))$. *In the case of the averaging estimator* $\hat{x}_{\text{ave}}$, *for all* $\epsilon \geq \underline{\eta}/n$, *the budget-optimal compensation contract is*

$$\pi(y_1, \ldots, y_n) = \left[\gamma(n-1)\epsilon + f(\eta^{-1}(n\epsilon))/\alpha\right] - \gamma\left(-y_i + \frac{1}{n}\sum_{j=1}^n y_j\right)^2,$$

with $\gamma = -(n/(n-1))^2 f'(\eta^{-1}(n\epsilon))/(\alpha\eta'(\eta^{-1}(n\epsilon)))$.

Note that a dual of the fundamental budget requirement in Proposition 3.2 can also be proved, in which the best estimation performance can be bounded for a given budget. In the next corollary, such a result is presented for the simplified symmetric game; the extension to non-symmetric cases follows the same line of reasoning as in Proposition 3.2.

Corollary 3.3 (Fundamental Performance Limit): *For any contract game such that* $\eta_i = \eta$, $f_i = f$, $\alpha_i = \alpha$ *for every i, the estimation quality of any ex ante individually rational compensation contract with the budget constraint* $B \leq B_{\max}$ *is lower bounded as* $\mathbb{E}\{\|x - \hat{x}_{\text{ave}}\|_2^2\} \geq \eta(f^{-1}(B_{\max}\alpha/n))/n$ *and* $\mathbb{E}\{\|x - \hat{x}_{\text{opt}}\|_2^2\} \geq V_{xx}/(1 + V_{xx}n/\eta(f^{-1}(B_{\max}\alpha/n)))$.

4. Numerical example

Consider a scenario in which sensors are asked to take the measurement of a variable, e.g. the temperature and humidity in a forest (to estimate the possibility of wild fires) or the travel time on a road. These battery-operated sensors can invest more energy to gather high-quality measurements. In addition, they may wish to take multiple samples, calculate the average, and transmit that average to the central planner. However, these approaches cost the sensors in terms of resources, i.e. time and energy consumption. Let the effort a for each sensor be proportional to a combination of their energy consumption and the amount of the time spent refining their measurement. Let $\eta_i(a) = \varrho/(\varrho + a)$ with $\varrho > 0$. This function captures the following: by investing more time, the sensors can gather more samples, and, hence, reduce the measurement error (which is inversely proportional to the number of the internal samples). Assume that the cost of that effort is given by $f_i(a) = \exp(\vartheta a)$ with $\vartheta > 0$. Such a function reflects the following: by spending more time, the sensors lose other opportunities and expend more battery charge, yet the corresponding gain in measurement quality may only yield modest improvement in return from the central planner. Also, for simplicity of presentation, consider a symmetric contract game.

First, consider the case where x is a zero-mean random variable with variance V_{xx}. Clearly, this numerical example satisfies the assumptions of Proposition 3.1, thus $B^{\min}_{\text{opt}}(n,\epsilon) = n\exp(\varrho\vartheta((V_{xx}-\epsilon)/(V_{xx}\epsilon n)-1))$. Here, we use the notation $B^{\min}_{\text{opt}}(n,\epsilon)$ to emphasise that the minimum required budget is a function of both n and ϵ. Using the mean value theorem (Rudin, 1976, p. 108), it can be deduced that there exists $\zeta \in [n, n+1]$ such that

$$B^{\min}_{\text{opt}}(n+1,\epsilon) - B^{\min}_{\text{opt}}(n,\epsilon) = \frac{\mathrm{d}B^{\min}_{\text{ave}}(\zeta,\epsilon)}{\mathrm{d}\zeta}$$
$$= \frac{V_{xx}\epsilon\zeta - \varrho\vartheta(V_{xx}-\epsilon)}{\epsilon V_{xx}\zeta^2} B^{\min}_{\text{ave}}(\zeta,\epsilon).$$

Thus, if $V_{xx}\epsilon n \geq \varrho\vartheta(V_{xx}-\epsilon)$, $B^{\min}_{\text{ave}}(n+1,\epsilon) \geq B^{\min}_{\text{ave}}(n,\epsilon)$ because $\zeta \geq n$. Alternatively, if $V_{xx}\epsilon(n+1) \leq \varrho\vartheta(V_{xx}-\epsilon)$, $B^{\min}_{\text{ave}}(n+1,\epsilon) \leq B^{\min}_{\text{ave}}(n,\epsilon)$ because $\zeta \leq n+1$. This shows that the optimal number of sensors (for minimising the minimum required budget for a given performance level) is the integer number to $[\varrho\vartheta(1/\epsilon - 1/V_{xx}) - 1, \varrho\vartheta(1/\epsilon - 1/V_{xx})]$, i.e. $\lfloor \varrho\vartheta(1/\epsilon - 1/V_{xx})\rfloor$. Similarly, for the case where x is deterministic, Proposition 3.2 shows that $B^{\min}_{\text{ave}}(n,\epsilon) = n\exp(\varrho\vartheta(1/(\epsilon n)-1))$. Again, using the mean value theorem, it can be deduced that there exists $\zeta \in [n, n+1]$ such that

$$B^{\min}_{\text{ave}}(n+1,\epsilon) - B^{\min}_{\text{ave}}(n,\epsilon) = \frac{\epsilon\zeta - \varrho\vartheta}{\epsilon\zeta^2} B^{\min}_{\text{ave}}(\zeta,\epsilon).$$

Following a similar reasoning, it can be deduced that the optimal number of sensors is given by $\lfloor \varrho\vartheta/\epsilon \rfloor$.

Figure 1 illustrates the minimum required budget $B^{\min}_{\text{ave}}(n,\epsilon)$ for achieving performance level ϵ versus n. The vertical dashed lines demonstrate the optimal number of sensors. As expected, up to a threshold, having access to more sensors reduces the budget but the trend reverses after the threshold.

To understand the magnitude of difference between the averaging estimator and the optimal linear estimator, define $\psi(n,\epsilon) := B^{\min}_{\text{ave}}/B^{\min}_{\text{opt}} - 1$ (in the case both estimators are used for stochastic x). Figure 2 illustrates $\psi(n,\epsilon)$ versus n and ϵ.

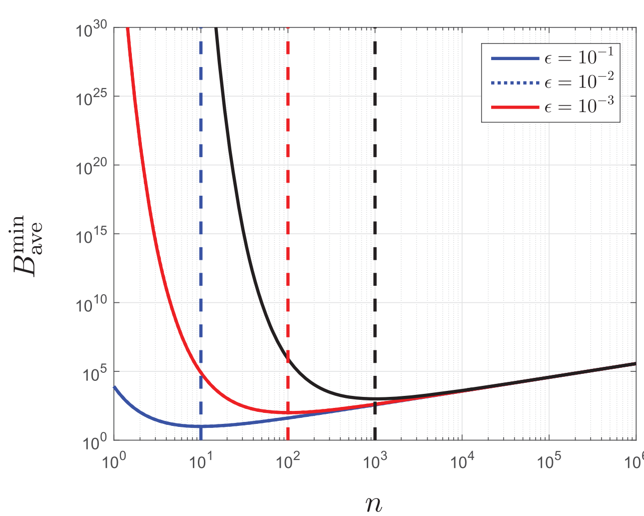

Figure 1. The minimum required budget for achieving performance level ϵ versus n. The vertical dashed lines demonstrate the optimal number of sensors.

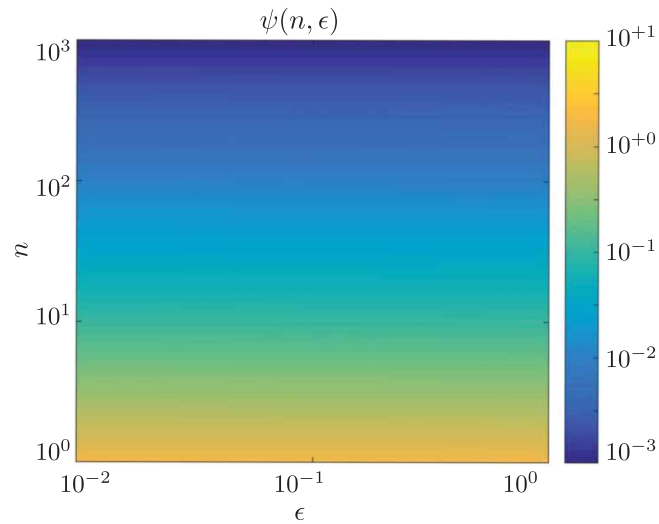

Figure 2. $\psi(n,\epsilon)$ versus n and ϵ.

Clearly, the difference between the two estimators is only visible in very low number of sensors (less than 10). Therefore, in the presence of large number of sensors, the receiver can use a much simpler estimator with a negligible financial loss.

5. Conclusions and future work

The behaviour of effort-averse sensors in response to long-term compensations is studied from the perspective of obtaining high-quality measurements. The interaction between the central planner employing an averaging based estimator and sensors via a contract is studied using a game. Optimal contracts, in terms of the budget, are constructed for achieving a specified level of estimation quality. Future work can focus on multiple estimators competing with each other for the attention of the sensors. This case is relevant to online markets for data acquisition and task completion, e.g. Amazon Mechanical Turk. Another avenue for future work is to investigate the effect of effort-averse sensors in the estimation of the linear dynamic system. This problem is intimately related to the problem of optimal event-triggered estimation. In the event-triggered literature (Lemmon, 2010), the sensors report their measurements to the operator only after a condition has been met (i.e. an event has been triggered) to save their resources. However, the results in this field provide sufficient conditions which, if met, guarantee that estimation or control with a required performance is possible while the optimality of the triggering rule in terms of resource consumption has not been studied.

Notes

1. The term effort averse is borrowed from the economics literature. For instance, in the insurance industry, it is observed that 'flat wage leaves the agents with no incentive to avoid behaviours ... that increase the risks' since such avoidance requires investing effort (which is clearly not rewarded under flat wages) (Ménard & Shirley, 2008, p. 352). Therefore, several studies have been devoted to designing optimal contracts that reward efforts either directly (based on the time and energy spent) or indirectly (based on the outcome) (Christensen & Feltham, 2005; Ménard & Shirley, 2008).

2. The scalar nature of the variable of interest is without loss of generality as the entries of a vector can individually be treated in a similar manner.
3. This can be determined by surveying the sensors or utilising historical data. Alternatively, the parameter can be eliminated by replacing it with the average value for the society; however, this results in an approximate analysis.

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